Throughput Analysis of Single-User and Multi-User MIMO with multiple antennas for Delay due to Partial CSI and Channel Quantization

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Abstract- This paper proposes an adaptive multiple-input-multiple-output (MIMO) transmission algorithm using imperfect channel state information (CSI) at the transmitter. This approach switches between single-user and multi-user MIMO modes for improvement of the spectral efficiency, based on the average signal-to-noise ratio (SNR), the amount of delay, and the channel quantization codebook size. The achievable ergodic rates for both single-user and multi-user MIMO transmissions with different kinds of channel information are derived, which are used to calculate mode switching points. It is shown that single-user MIMO is relatively robust to imperfect channel information, while multi-user MIMO with Block-Diagonalization precoding loses spatial multiplexing gain with a fixed delay or fixed codebook size. With imperfect channel information, as SNR increases, there are two mode switching points. The single-user mode is selected in both low and high SNR regimes, due to array gain and its robustness to the channel information error, respectively. The operating regions for single-user and multi-user modes with different delays and codebook sizes are determined, which can be used to select the preferred mode. It is shown that the multi-user mode may never be activated when the delay is large or the codebook size is small.

Keywords- MIMO, Single-user MIMO, Multi-user MIMO, channel state information at the transmitter, SVD,BD.

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1 INTRODUCTION

Multiple-input multiple-output (MIMO) transmission is a promising technique for higher data rate wireless communications [1] [2]. Over the last decade, the point-to-point MIMO link (SU-MIMO) has been extensively researched and has transited from a theoretical concept to a practical technique. Due to space and complexity constraints, however, current mobile terminals only have one or two antennas, which limit the performance of the SU-MIMO link. Multi-user MIMO (MU-MIMO) provides the opportunity to overcome such a limitation by communicating with multiple mobiles simultaneously. It effectively Increases the number of equivalent spatial channels and provides spatial multiplexing gain proportional to the number of transmit antennas at the base station. There are many technical challenges that must be overcome to exploit the full benefits of MU-MIMO. A major one is the requirement of channel state information (CSI) at the base station, which is difficult especially for the downlink/broadcast channel. Limited feedback [3]-[5], where quantized channel information is provided to the transmitter via a low-rate feedback channel, is one solution to this problem. Besides quantization, there are other imperfections in the available CSI at the base station, such as estimation error and feedback delay. Cumulatively, such imperfections will greatly degrade the performance of MU-MIMO. With imperfect CSI, it is not clear whether–or more to the point, when–MU-MIMO can outperform SU-MIMO. In this paper, we propose to switch between SU and MU-MIMO modes based on the achievable rate of each technique with practical receiver assumptions.

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SU-MIMO communication has been well studied. With full CSI at the receiver (CSIR), the capacity of SU-MIMO grows linearly with the minimum number of transmit and receive antennas in the Rayleigh fading channel even with only channel distribution information at the transmitter [1]. Partial or full CSI at the transmitter (CSIT) can

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improve performance and reduce complexity. Recently, limited feedback techniques have been developed to exploit this performance gain for different transmission techniques [6]–[7]. With estimation errors at the transmitter and receiver, the capacity bounds optimal power allocation strategies for SU-MIMO were derived in [8]. For the MIMO broadcast channel, it has been shown that dirty-paper coding (DPC) [9] is optimal [10], but in practice it is difficult to implement. Therefore, low-complexity transmission techniques that are able to approach the optimal performance are required. In this paper, we consider both CSI delay and channel quantization, and propose to switch between SU and MU modes based on the achievable rates.

The main purposes of this paper are as follows.

SU vs. MU Analysis. We investigate the impact of imperfect CSIT due to delay and channel quantization. We show that the SU mode is more robust to imperfect CSIT, while MU-MIMO suffers severe performance degradation due to residual inter-user interference and loses spatial multiplexing gain with a fixed delay or codebook size.

*Mode Switching Algorithm***.** Mode switching between SU and MU modes is proposed based on ergodic sum rate as a function of the average SNR, normalized Doppler frequency, and the quantization codebook size. Tight closed-form bounds/ approximations give the mode switching points.

*Operating Regions***.** The operating regions for SU and MU modes are determined. With a fixed delay and codebook size, if the MU mode is possible at all, there are two mode switching points, with the SU mode preferred in both low and high SNR regimes. When delay is large, or the codebook size is small, the MU mode may never be activated.

2 SYSTEM MODEL

We consider a MIMO downlink system, where the transmitter (base station) has N_t antennas and each mobile user has N_r antennas. The system parameters are listed in Table 1.

TABLE 1: System Parameters

Symbol	Description
Nt	number of transmit antennas at the base station
N_r	number of receive antennas at each mobile
U	number of mobile users
B	number of feedback bits
L	quantization codebook size, $L = 2^B$
Γ	average SNR
N	time index
D	CSI delay in the number of symbols
T_s	the length of each symbol
T	the length of delay, $\tau = T_s D$
f_d	the Doppler frequency

During each transmission period, the transmitter sends symbols to one (SU-MIMO mode) or multiple (MU-MIMO mode) users. The received signal at the u-th user at time n is given as

$$\mathbf{Y}_u[n] = \mathbf{H}_u[n]\mathbf{T}[n]\mathbf{X}[n] + \mathbf{Z}_u[n] \qquad (1)$$

where $\mathbf{H}_u[n]$ is the $Nr \times Nt$ channel matrix from the transmitter to the u^{th} user, and $\mathbf{Z}_u[n]$ is the normalized complex Gaussian noise vector with entries distributed according to CN(0,1). For the SU mode, **X**[n] is the transmit signal for the u^{th} user, and **T**[n] is the transmit precoder for the u^{th} user. For the MU mode, **X** [n] is the aggregated signal

$$X[n] = \left[X_1^*[n], X_2^*[n], \ldots X_U^*[n]\right]^*,$$

and **T**[n] is the aggregated precoding matrix **T**[n] = [T1[n],T2[n], · · · ,T_U[n]]. The transmit power constraint is **E**{x*[n]x[n]} = γ. As the noise is normalized, γ is also the value of the average SNR. To assist the analysis, we assume that the channel Hu[n] is well modeled as a spatially white Gaussian channel, with entries hi,j [n]~CN(0, 1). The results will be different for different channel models..

The objective in this paper is to study the impact of imperfect CSIT, we assume perfect CSIR and the transmitter acts as if the CSIT is perfect. We study the following three CSIT scenarios.

2.1 *Perfect CSIT:* this is the baseline case where the transmitter has perfect CSI.

2.2 *TDD System:* for a TDD system, the transmitter can estimate the CSI through channel reciprocity. We assume such estimation is perfect. There will be delay in the available CSIT, however, due to propagation /processing/ protocol.

2.3 *FDD System*: the receiver quantizes the CSI and feeds back to the transmitter through a dedicated Low -rate feedback channel, so there are both CSI delay and channel quantization errors.

2.1 CSI Delay Model

We consider a time-varying channel, where the channel stays constant for a symbol duration and changes from symbol to symbol according to a stationary correlation model. There is a delay of D symbols in the available CSIT. The current channel H[n] and its delayed version $\mathbf{H}[\mathbf{n} - D]$ are jointly Gaussian with zero mean and are related in the following manner

$$\mathbf{H}[n] = \rho \mathbf{H}[n-D] + \mathbf{E}[n] \tag{2}$$

Where $\boldsymbol{E}$[n] is the channel error matrix, with i.i.d. entries $e_{ij}[\mathbf{n}] \sim CN(0, \epsilon_e^2)$, and it is uncorrelated with $\boldsymbol{H}[n - D]$. For the numerical analysis, the classical Clarke's isotropic scattering model will be used as an example, for which the correlation coefficient is $\rho(D) = J_0(\pi f_d D T_s)$ with Doppler spread f_d [13], where $J_0(\cdot)$ is the zero-th order Bessel function of the first kind, and $\epsilon_e^2(D) = 1 - \rho^2(D)$. Therefore, both $\rho(D)$ and $\epsilon_e(D)$ are determined by the normalized Doppler frequency $f_d D T_s$. In the following, we will denote them as ρ and ϵ_e for simplicity. This is a noiseless delayed feedback model used in [6].

2.2 Channel Quantization

For FDD systems, limited feedback techniques can provide partial CSIT through a dedicated feedback channel from the receiver to the transmitter. The channel information is fed back using a quantization codebook known at both the transmitter and receiver. Channel direction information is fed back for the precoder design, which is discussed in this section. The received SINR is assumed to be known perfectly to determine the transmit rate.

For $\mathbf{N}_r > 1$, the quantization codebook C_u for user u consists of 2^B matrices that satisfy $C_l C_l^* = I_{N_r}, \forall C_l \in C_u$. The index of the quantized channel is chosen as

$$I_u = \arg \min_{1 \le l \le L} d^2(H_u, C_{u,l}) \tag{3}$$

The *chordal distance* is used as the distance metric [8]. The quantization distortion is defined as

$$\overline{D} = \mathrm{E}\left[\min_{C \in C_u} d^2(H_u, C)\right] \tag{4}$$

With RVQ, it was shown in [11] that

$$D \le \frac{\Gamma\left(\frac{1}{T}\right)}{T} \left(C_{N_t N_r}\right)^{-\frac{1}{T}} 2^{-\frac{B}{T}} + N_r \exp\left[-\left(2^B C_{N_t N_r}\right)^{1-a}\right] \tag{5}$$

where $\mathbf{T} = \boldsymbol{N}_r (\boldsymbol{N}_t - \boldsymbol{N}_r)$ and $a \in (0,1)$ is a real number between 0 and 1 chosen such that $\left(2^B C_{N_t N_r}\right) \le 1$, with $C_{N_r N_t} = \frac{1}{T!} \prod_{i=1}^{N_r} \frac{(N_t - i)!}{(N_r - i)!}$.

3 PERFECT CSIT

3.1 *SVD (Singular Value Decomposition) Transceiver(SU-MIMO)*

With perfect CSIT, the SVD transceiver is optimal for the point-to-point MIMO link [8]. Denote the SVD of the channel matrix as $H[n]=U[n]\Lambda[n]V^*[n]$. The precoding matrix is $V[n]$ while the decoding matrix is $U[n]$. The optimal input is $\Phi = \mathrm{E}[X[n]X^*[n]] = V[n]P[n]V^*[n]$, with power loading matrix $P[n]$ obtained through water-filling power allocation. The asymptotic results for such a system was presented in [12], as $N_t, N_r \to \infty$ with $\frac{N_t}{N_r} \to \beta$, which is valid only for a certain range of SNR due to the water-filling power allocation. Recall that the capacity gain of the MIMO system with full CSIT over that with channel distribution information at the transmitter (CDIT) comes from a power gain of $\frac{N_t}{N_r}$ for **N**t >**N**r , as with CSIT the transmit power can be focused on *N*r non-zero eigenmodes [9]. To assist the analysis of mode switching, we approximate the capacity with perfect CSIT by the one with CDIT but with $\gamma\frac{N_t}{N_r}$ as the effective SNR. With CDIT and perfect CSIR, the optimal input covariance for i.i.d. Rayleigh fading channel is $\Phi = \frac{\gamma}{N_t} I_{N_r}$ with Gaussian distribution [9]. The ergodic capacity for this system is then given by

$$C_{iso} = \mathrm{E}\left[\log\det\left(I_{N_r} + \frac{\gamma}{N_t} H[n]H^*[n]\right)\right] \quad (6)$$

The following theorem gives the asymptotic result for this capacity as $N_t, N_r \to \infty$ with $\frac{N_t}{N_r} \to \beta$.

Theorem 5: For a point-to-point MIMO link with CDIT and full CSIR, the asymptotic capacity per receive antenna is [28]

$$\frac{C_{iso}(\beta,\gamma)}{N_r} = \log_2\left[1+\gamma - F\left(\beta, \frac{\gamma}{\beta}\right)\right] + \beta\log_2\left[1+\frac{\gamma}{\beta} - \right. \quad (7)$$

With

$$F(x,y) = \frac{1}{4}\left[\sqrt{1+y\left(1+\sqrt{x}\right)^2} - \sqrt{1+y\left(1-\sqrt{x}\right)^2}\right]^2$$

Therefore, ergodic capacity of the SVD system with perfect CSIT is approximated as

$$C_{SVD} \approx \mathrm{E}\left[\log\det\left(I_{N_r} + \frac{\beta\gamma}{N_t} H[n]H^*[n]\right)\right] = C_{iso}(\quad (8)$$

This is easy to calculate and valid for the whole SNR range. In addition, it will be shown later that this approximation is accurate.

3.2 BD (Block Diagonalization) System(MU-MIMO)

With perfect CSIT and assuming the channels are sufficiently different, BD precoding Achieves $H_u[n]T_{u'}[n] = 0, \forall u' \neq u$. To assist the analysis, $T_u[n]$ is constrained to be a unitary matrix, i.e. $T_u^*[n]T_u[n] = I_{N_r}$, which is consistent with the uniform power allocation assumed in this paper. In this way, user u sees an $N_r \times N_r$ effective point-to-point interference-free channel. Its received signal becomes

$$y_u[n] = H_u[n]T_u[n]x_u[n] + z_u[n] = H_{eff,u}[n]x_u[n] + \quad (9)$$

where $H_{eff,u}[n] = H_u[n]T_u[n]$. As $T_u[n]$is a unitary matrix, which is independent of $H_u[n]$, $H_{eff,u}[n]$ is also a complex Gaussian matrix as $H[n]$, i.e $H_{eff,u}[n] \sim CN(0_{N_r \times N_r}, N_r I_{N_r})$.

Assuming the number of data streams for user u is equal to the number of receive antennas of us u, and with equal power allocation, the input covariance is $\Phi_{BD,u} = \frac{\gamma}{N_t} I_{N_r}$. The achievable ergodic rate for the u^{th} user is given by

$$R_{BD,u} = \mathrm{E}\left[\log_2 \det\left(I_{N_r} + \frac{\gamma}{N_t} H_{eff,u}[n] H^*_{eff,u}[n]\right)\right] \quad (10)$$

which is similar to (10), but with an N_r x N_r effective channel. Therefore, the achievable rate for the BD system with perfect CSIT is given as

$$R_{BD} = \sum_{u=1}^{U} R_{BD,u} \approx U N_r C_{iso}(1, \gamma/\beta) \quad (11)$$

The spatial multiplexing gain of BD is ***UN***r, compared to ***N***r for the SVD system.

3.3 Mode Switching Point

Equating (6) and (8), the approximation of the mode switching point for SU and MU modes can be calculated. Some numerical results are shown in Fig. 1. We see that the asymptotic approximation is good for SVD and is slightly loose for BD system at high SNR, due to the effective low-dimension 2 × 2 channels for BD users. However, it accurately predicts the switching point. With perfect CSIT, there is only one switching point. The SVD mode is active at low SNR due to its array gain; the BD mode is preferred at high SNR due to its enhanced spatial multiplexing gain.

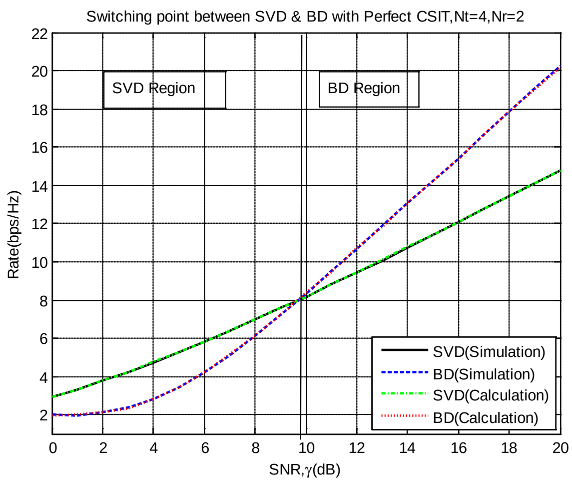

Figure 1 - Switching point between SVD and BD modes with perfect CSIT,N_t=4,N_r=2.

4 TDD SYSTEM-CSI DELAY

4.1 SVD Transceiver

With CSI delay, the SVD transceiver based on the outdated channel cannot perfectly diagonalize the channel matrix. The receiver performs joint decoding rather than separate decoding which is only possible with perfect CSIT. Therefore, the achievable rate is

$$R_{SVD}^{(D)} = \mathrm{E}\left[\log\det\left(\mathrm{I}_{N_r} + H[n]\Phi[n-D]H^*[n]\right)\right] \quad (12)$$

where $\Phi[n-D]$ is the input covariance based on the outdated CSIT. The receiver estimates this achievable rate based on the pilot symbol and feeds back it to the transmitter.

4.2 BD System

With delayed CSIT, BD precoding matrices cannot perfectly cancel mutual interference, as they only achieve $H_u[n-D]T_{u'}^{(D)}[n] = 0, \forall u' \neq u$. The received signal for user u thus becomes

$$y_u[n] = H_{eff,u}[n]X_u[n] + \mathrm{E}_u[n]\sum_{u \neq u'} T_{u'}^{(D)}[n]X_{u'}[n] + \quad (13)$$

Treating residual interference as noise, the achievable rate for the u^{th} user is given by

$$R_{BD,u}^{(D)} = \mathrm{E}\left[\log_2\det\left(\mathrm{I}_{N_r} + \frac{\gamma}{N_t}H_{eff,u}[n]H_{eff,u}^*[n]R_u^{-}\right.\right. \quad (14)$$

Where $\mathbf{R}_u[n]$ is the interference-plus-noise covariance matrix, given by

$$R_u[n] = \mathrm{E}_u[n]\mathrm{E}\left[\sum_{u' \neq u} T_{u'}^{(D)}[n]X_{u'}[n]X_{u'}^*[n]T_{u'}^{(D)*}[n]\right]\mathrm{E} \quad (15)$$

$$= \mathrm{E}_u[n]\left[\sum_{u' \neq u}\frac{\gamma}{N_t}T_{u'}^{(D)}[n]T_{u'}^{(D)*}[n]\right]\mathrm{E}_u^*[n] + \mathrm{I}_{N_r}$$

To calculate (15), we first focus on an upper bound for the rate loss, as did in [11] for the limited feedback system. This will give a lower bound for (15) and provide useful insights. Then for a 2-user special case, we will provide asymptotic results which are accurate to get the mode switching points. The following theorem provides an upper bound for the rate loss of the BD system with delay.

Theorem 1: Compared to the system with perfect CSIT, the rate loss for the u^{th} user of the delayed BD system is upper bounded by [12]

$$\Delta R_{BD,u}^{(D)} = R_{BD,u} - R_{BD,u}^{(D)} \le N_r \log_2\left[\left(N_t - N_r\right)\frac{\gamma}{N_t}\epsilon\right. \quad (16)$$

Remark 1: The rate loss for each user is proportional to the number of receive antennas, which equals the number of data streams, so the user with more data streams will have a larger rate loss. There is a scaling factor $N_t - N_r$ on $\in_{e,u}^2$, which is the number of interfering data streams. Equivalently, there are $N_t - N_r$ *virtual interfering single-antenna users* with the equivalent SNR as $\frac{\gamma}{N_t}\in_{e,u}^2$. Therefore, the rate loss for each data stream of BD is lower, as BD has fewer virtual interfering users, which means BD is more robust to the CSI delay. Intuitively, this is because the receiver of the BD system can perform joint decoding and there is no interference between data streams for the same user. Next, we consider a two-user BD system and provide asymptotic results for the achievable rate. Without loss of generality, we focus on user 1. We denote γ1 and γ2 as SNRs for user 1 and user 2, and $N_{r,1}$ and $N_{r,2}$ as the numbers of receive antennas for user 1 and user 2.

Theorem 2: For a 2-user BD system with CSI delay, the achievable rate per receive antenna for user 1 is [12]

$$\frac{R_{BD,1}^{(D)}}{N_{r,1}} = \frac{N_{r,2}}{N_{r,1}}\log_2\left(\frac{\frac{N_{r,2}}{N_{r,1}} + \eta 1\gamma 2\in_{e,1}^2}{\frac{N_{r,2}}{N_{r,1}} + \eta 2\gamma 2\in_{e,1}^2}\right) + \log_2\left(1+\eta 1\gamma 1\right) \quad (17)$$

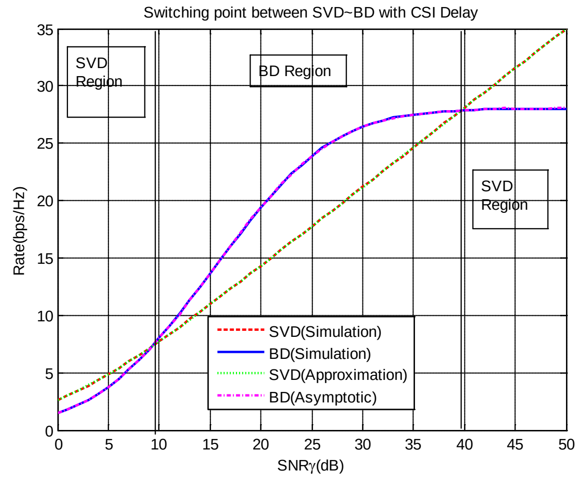

Figure 2 - Switching point between SVD and BD modes with CSI delay, N_t=4, f_c=2GHz, τ =0.001 sec and terminal speed is 10 km/hr.

with η_1 and η_2 as the positive solutions to

$$\eta_1 + \frac{\gamma_1\eta_1}{\gamma_1\eta_1 + 1} + \frac{N_{r,2}}{N_{r,1}}\frac{\eta_1\gamma_2\in_{e,1}^2}{\eta_1\gamma_2\in_{e,1}^2 + \frac{N_{r,2}}{N_{r,1}}} = 1,$$

$$\eta_2 + \frac{N_{r,2}}{N_{r,1}}\frac{\eta_2\gamma_2\in_{e,1}^2}{\eta_2\gamma_2\in_{e,1}^2 + \frac{N_{r,2}}{N_{r,1}}} = 1$$

Such asymptotic results can be used to accurately predict the mode switching point.

4.3 Mode Switching Point

An example of mode switching for SVD and BD systems is shown in Fig. 2. We see that the asymptotic result from above *Theorem 2* is also useful in the non-

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asymptotic region. There are two switching points. Due to delay the SVD mode is preferred at both low and high SNR and BD is preferred at medium SNR. We have consider here terminal speed 10 km/hr.

5 FDD SYSTEM–CSI DELAY AND CHANNEL QUANTIZATION

5.1 SVD Transceiver

For SVD transceiver of the FDD system, the receiver will feed back the index of a precoding matrix to the transmitter. Given **H**[n] and the precoding matrix **F**[n], the mutual information is

$$I(F) = \log_2 \det\left(\mathrm{I}_{N_r} + \frac{\gamma}{N_r} H[n]F[n]F^*[n]H^*[n] \right) \quad (18)$$

Let $\overline{V}_R[n]$ denote the first N_r columns of **V**[n], which is the right singular matrix of **H**[n]. As shown in [10], the unitary precoding matrix that maximizes the mutual information *I*(F) is $F_{opt} = \overline{V}[n]$. With CSI delay, the available precoding matrix, $\hat{F}[n]$, at the transmitter is based on **H**[n − *D*]. The achievable rate of this system is

$$R_{SVD}^{(QD)} = \mathrm{E}\left[\log\det\left(\mathrm{I}_{N_r} + H[n]\hat{F}[n]\hat{F}^*[n]H^*[n]\right)\right] \quad (19)$$

This is difficult to calculate. Similar to the delayed system, we will approximate the capacity with perfect CSIT, which will be verified by numerical results.

5.2 BD System

For the BD system with both CSI delay and channel quantization, the feedback information is the quantization of the channel direction matrix, $\hat{H}_u[n]$. Therefore, BD precoding matrices are designed based on $\hat{H}_u[n-D]$, which achieves $\hat{H}_u[n-D]T_{u'}^{(QD)}[n] = 0, \forall u' \neq u.$

The achievable rate for the u^{th} user is given by[13]

$$R_{BD,u}^{(QD)} = \mathrm{E}\left[\log_2 \det\left(\mathrm{I}_{N_r} + \frac{\gamma}{N_t} H_{eff,u}[n]H_{eff,u}^*[n]\hat{R}_u^{-}\right.\right. \quad (20)$$

where $\hat{R}_u[n]$ is given by

$$\hat{R}_u[n] = H_u[n]\left[\sum_{u' \neq u} \frac{\gamma}{N_t} T_{u'}^{(QD)}[n]T_{u'}^{(QD)*}[n]\right]H_u^*[n] + \mathrm{I} \quad (21)$$

The rate loss for such a system is given in the following theorem.

Theorem 3: With both CSI delay and channel quantization, the rate loss for the u^{th} user of the BD system is given by[13]

$$\Delta R_{BD,u}^{(QD)} = R_{BD,u} - R_{BD,u}^{(QD)} \leq N_r \log_2\left[(N_t - N_r)\frac{\gamma}{N_t}\left(\right. \right. \quad (22)$$

where $\overline{D}$ is the distortion of channel quantization, defined in (4).

Remark 2: Theorem 3 explicitly shows the impact of quantization and delay on the BD system, through the terms of $\overline{D}$ and $\in_{e,u}^2$, respectively. Equivalently, there are $N_t - N_r$ *virtual interfering single-antenna users* with the equivalent SNR as $\frac{\gamma}{N_t}\left(\rho_u^2 \frac{N_t}{N_t - N_r}.\frac{\overline{D}}{N_r} + \in_{e,u}^2\right)$. With a fixed codebook size or delay, the system is interference-limited.

5.3 Mode Switching Point

For BD with both delay and quantization, a lower bound of the achievable rate is provided in *Theorem 8*, but such a lower bound based on rate loss analysis is very loose. Analogous to the bound in (20) for the ZF system, we provide the following lower bound to calculate the mode switching point [14]

$$R_{BD,u}^{(QD)} \geq \mathrm{E}\left[\log_2 \det\left(\mathrm{I}_{N_r} + \frac{\gamma}{\Delta_{BD,u}^{(QD)} N_t} H_{eff,u}[n] H_{eff,u}^{*} \right| \right. \quad (23)$$

Where $\Delta_{BD,u}^{(QD)} = \left[\left(\frac{\rho_u^2 D}{N_r} + \frac{N_t - N_r}{N_t} \in_{e,u}^2\right)\gamma + 1\right]$ is from *Theorem 3*. Essentially, this is to approximate $\hat{R}_u[n]$ by $\Delta_{BD,u}^{(QD)} \mathrm{I}_{N_r}$, i.e. to approximate inter-user interference as white Gaussian noise. This lower bound can be calculated with the asymptotic result shows an example of the achievable rates for SVD and BD systems with both delay and quantization. Note that the mobility is very low (v = 1 km/hr), and the codebook size is large (**B** = 18), but BD can not outperform SVD over the whole SNR range, while the performance of SVD is very close to the case with perfect CSIT. It is also shown that the approximation (22) can be used to predict the mode switching point, while the lower bound from *Theorem 3* is very loose. The approximation for SVD is very accurate.

The operating regions for SVD and BD modes are plotted in Fig. 3. , we see that the operating region for BD is much smaller. This is because that the SVD system provides both spatial multiplexing gain and power gain, so its operating region is relative large.

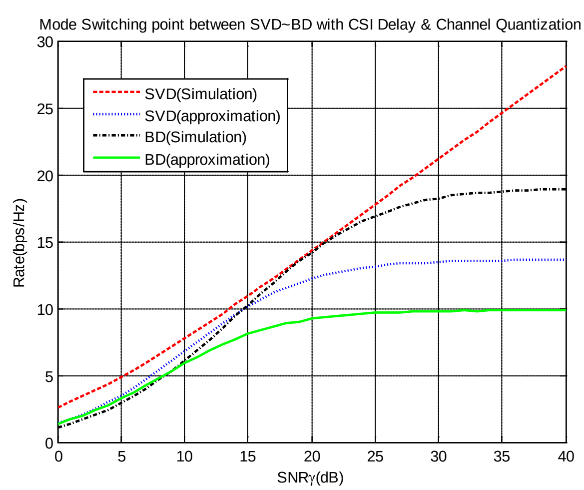

Figure 3 - Mode Switching between SVD and BD modes with CSI delay and channel quantization, B=10,N_t=4,f_c=2GHz, τ =0.001 sec and terminal speed is 1 km/hr**.**

Therefore, we would expect that in most of the practical scenarios with $N_r > 1$ the system will operate in the SVD mode.

6 CONCLUSIONS

We have derived the achievable ergodic rates of SU and MU-MIMO systems with imperfect CSIT, due to delay and channel quantization. It is shown that SU-MIMO is relatively more robust to imperfect CSIT. For MU-MIMO, there are equivalently a number of virtual interfering users due to residual interference with CSI error, which makes the system interference-limited. A mode switching algorithm between SU and MU modes is proposed to improve the spectral efficiency. Mode switching points are calculated, which is based on the average SNR, normalized Doppler frequency, and the codebook size. With a fixed delay and codebook size, MU mode is only possible to be active in the medium SNR regime. When delay is large or the codebook size is small the MU mode may never be activated.

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REFERENCES

[1] I. E. Telatar, "Capacity of mulit-antenna Gaussian channels," *Europ. Trans. Telecommun.*, vol. 10, pp. 585–595, Nov. 1999.
[2] A. Goldsmith, S. A. Jafar, N. Jindal, and S. Vishwanath, "Capacity limits of MIMO channels," *IEEE J. Select. Areas Commun.*, vol. 51, no. 6, pp. 684–702, Jun. 2003.
[3] Mrutyunjaya Panda and Santosh Kumar Acharya" SU *vs.* MU MIMO Selection for Perfect CSIT and CSI Delay Conditions", *SIT, Bhubaneswar, NCICT-2011*, pp.101-104, 10-11 September, 2011.
[4] T. Yoo, N. Jindal, and A. Goldsmith, "Multi-antenna downlink channels with limited feedback and user selection," *IEEE J. Select. Areas Commun.*, vol. 25, no. 7, pp. 1478–1491, Sept. 2007.
[5] G. Caire, N. Jindal, M. Kobayashi, and N. Ravindran, "Multiuser MIMO downlink made practical: Achievable rates with simple channel state estimation and feedback schemes," *Submitted to IEEE Trans. Information Theory*, vol. Available online at http://arxiv.org/abs/0711.2642.
[6] N. Ravindran and N. Jindal, "Limited feedback-based block diagonalization for the MIMO broadcast channel," *to appear, IEEE Journal Sel. Areas in Communications*, 2008.
29
[7] K. Mukkavilli, A. Sabharwal, E. Erkip, and B. Aazhang, "On beamforming with finite rate feedback in multiple-antenna systems," *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2562–2579, Oct. 2003.
[8] D. J. Love, R. W. Heath, Jr., W. Santipach, and M. Honig, "What is the value of limited feedback for MIMO channels?" *IEEE Commun. Mag.*, vol. 42, no. 10, pp. 54–59, Oct. 2004.
[9] T. Yoo and A. Goldsmith, "Capacity and power allocation for fading MIMO channels with channel estimation error," *IEEE Trans. Inform. Theory*, vol. 52, no. 5, pp. 2203–2214, May 2006.
[10] M. Costa, "Writing on dirty paper," *IEEE Trans. Inform. Theory*, vol. 39, no. 3, pp. 439–441, May 1983.
[11] S. Vishwanath, N. Jindal, and A. Goldsmith, "Duality, achievable rates, and sum-rate capacity of MIMO broadcast channels," *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2658–2668, Oct. 2003.
[12] Q. H. Spencer, A. L. Swindlehurst, and M. Haardt, "Zero-forcing methods for downlink spatial multiplexing in multi-user MIMO channels," *IEEE Trans. Signal Processing*, vol. 52, no. 2, pp. 461–471, Feb. 2004.
[13] J. Lee and N. Jindal, "High SNR analysis for MIMO broadcast channels: Dirty paper coding vs. linear precoding," *IEEE Trans. Inform. Theory*, vol. 53, no. 12, pp. 4787–4792, Dec. 2007.
[14] R. H. Clarke, "A statistical theory of mobile radio reception," *Bell System Tech. J.*, vol. 47, pp. 957–1000, 1968.